MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 3, 2015/2016

PCM0235 - CALCULUS

(Foundation in Information Technology/ Foundation in Life Sciences)

3 JUNE 2016
3.00 p.m – 5.00 p.m
(2 Hours)

INSTRUCTIONS TO STUDENT

1. This Question paper consists of 2 pages excluding the cover page and appendix.
2. Answer **FIVE** out of **SIX** questions. All questions carry equal marks and the distribution of the marks for each question is given.
3. Write all your answers in the Answer Booklet provided.

Answer any five questions.

Question 1 (10 marks)

a) Evaluate the following limits.

i. $\lim_{x\to 1}\dfrac{\sqrt{x+3}-2}{x-1}$ (3 marks)

ii. $\lim_{x\to -\infty}\dfrac{2x^4+3x^2-10x}{3x^5+6x}$ (2 marks)

iii. $\lim_{x\to 0}\dfrac{2x+\sin 4x}{x}$ (3 marks)

b) Find the value of the constant, k such that the function $f(x)=\begin{cases} kx^2, & x\le 2 \\ 2x+k, & x>2 \end{cases}$ is continuous for all real numbers. (2 marks)

Question 2 (10 marks)

a) Find $\dfrac{dy}{dx}$ for $y=\dfrac{e^{x^3}}{x^3}$. Simplify the answer. (3 marks)

b) Use logarithmic differentiation to find $\dfrac{dy}{dx}$, given $y=x^{\sin x}$. Hence find the equation of the tangent line at $x=\pi$. (4 marks)

c) Given $f(x)=\ln(x^2+4)^2$, find $f'(0)+f''(0)$. (3 marks)

Question 3 (10 marks)

a) Given $y=x^4-8x^2+16$. Find the interval on which the function is increasing and the interval on which the function is decreasing. Identify the x-coordinates of any local maximum points and local minimum points. (6 marks)

b) Oil spilled from a ruptured tanker, spreads in a circle whose area increases at a constant rate 6 km^2/ hr. How fast is the radius of the spill increasing when the area is 12 km^2? (4 marks)

Continued...

Question 4 (10 marks)

a) Find:

i. $\int\left(6x+\frac{x^3-x^2}{x-1}\right)dx$. (3 marks)

ii. $\int \sin 2x \cos^4 2x dx$. (3 marks)

b) Use integration by parts to evaluate $\int_1^e \frac{\ln x}{\sqrt{x}}dx$. (4 marks)

Question 5 (10 marks)

a) Find the area of the region bounded by $y = x^2 + 5, x = -2, x = 3$ and the x-axis. (4 marks)

b) Sketch and identify the region bounded by $y = (x-4)^2$ and $y = 9$. Hence, find the volume of the solid rotation when the region is rotated about the x–axis. (6 marks)

Question 6 (10 marks)

a) Solve the first order differential equation, $x\frac{dy}{dx} + y = e^x$ given $y(1) = 1$. (5 marks)

b) Solve the second order differential equation, $2y''+y'-6y = 0$ given $y(0) = 2$ and $y'(0) = 1$. (5 marks)

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APPENDIX

A. Differentiation Rules

$\frac{d}{dx}[x^n] = nx^{n-1}$; n is any real number

$\frac{d}{dx}[f(x).g(x)] = f(x)g'(x) + f'(x)g(x)$; The Product Rule

$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$; The Quotient Rule

$\frac{d}{dx}f(g(x)) = f'(g(x)).g'(x)$; The Chain Rule

$\frac{d}{dx}[g(x)]^n = n[g(x)]^{n-1}.g'(x)$; The power rule combined with the chain rule:

$\frac{d}{dx}[\sin x] = \cos x$ $\quad$ $\frac{d}{dx}[\cos x] = -\sin x$ $\quad$ $\frac{d}{dx}[\tan x] = \sec^2 x$

$\frac{d}{dx}[\sec x] = \sec x \tan x$ $\quad$ $\frac{d}{dx}[\cot x] = -\csc^2 x$ $\quad$ $\frac{d}{dx}[\csc x] = -\csc x \cot x$

$\frac{d}{dx}[e^x] = e^x$ $\quad$ $\frac{d}{dx}[\ln x] = \frac{1}{x}; \quad x > 0$

B. Basic Integration Formulas

$\int cf(x)\,dx = c\int f(x)\,dx$ $\quad$ $\int k.dx = kx + C$

$\int [f(x) \pm g(x)]dx = \int f(x)dx \pm \int g(x)dx$ $\quad$ $\int x^n dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1$

$\int e^x\,dx = e^x + C$ $\quad$ $\int \frac{1}{x}dx = \ln|x| + C$

Integration by-parts: $\int u\,dv = uv - \int v\,du$

Volume (disk) $= \pi\int_a^b (f(x))^2\,dx$ $\quad$ **Area** $= \int_a^b (f(x) - g(x))dx$

Volume (washer) $= \pi\int_a^b \left[(f(x))^2 - (g(x))^2\right]dx$